Decoherence control for optical qubits

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Abstract. Photons in cavities have been already used for the realization of simple quantum gates [Q.A. Turchette *et al.*, Phys. Rev. Lett. **75**, 4710 (1995)]. We present a method for combatting decoherence in this case.

1 Introduction

Quantum optics is usually concerned with the generation of nonclassical states of the electromagnetic field and their experimental detection. However with the recent rapid progress in the theory of quantum information processing the *protection* of quantum states and their quantum dynamics also is becoming a very important issue. In fact what makes quantum information processing much more attractive than its classical counterpart is its capability of using entangled states and of processing generic linear superpositions of input states. The entanglement between a pair of systems is capable of connecting two observers separated by a space-like interval, it can neither be copied nor eavesdropped on without disturbance, nor can it be used by itself to send a classical message [1]. The possibility of using linear superposition states has given rise to quantum computation, which is essentially equivalent to have massive parallel computation [2]. However all these applications crucially rely on the possibility of maintaining quantum coherence, that is, a defined phase relationship between the different components of linear superposition states, over long distances and for long times. This means that one has to minimize as much as possible the effects of the interaction of the quantum system with its environment and, in particular, decoherence, i.e., the rapid destruction of the phase relation between two quantum states of a system caused by the entanglement of these two states with two different states of the environment [3, 4].

Quantum optics is a natural candidate for the experimental implementation of quantum information processing systems, thanks to the recent achievements in the manipulation of single atoms, ions and single cavity modes. In fact two quantum gates have been already demonstrated [5, 6] in quantum optical systems and it would be very important to develop strategies capable of *controlling the decoherence* in experimental situations such as those described in Refs. [5, 6].

In this paper we propose a simple physical way to control decoherence and protect a given quantum state against the destructive effects of the interaction with the environment: applying an appropriate feedback. The feedback scheme considered here has a quantum nature, since it is based on the injection of an appropriately prepared atom in the cavity and some preliminary aspects of the scheme, and its performance, have been described in Refs. [7, 8]

2 A feedback loop for optical cavities

Applying a feedback loop to a quantum system means subjecting it to a series of measurements and then using the result of these measurements to modify the dynamics of the system. Very often the system is continuously monitored and the associated feedback scheme provides a continuous control of the quantum dynamics. An example is the measurement of an optical field mode, such as photodetection and homodyne measurements, and for these cases, Wiseman and Milburn have developed a quantum theory of continuous feedback [9]. This theory has been applied in Refs. [10] to show that homodyne-mediated feedback can be used to slow down the decoherence of a Schrödinger cat state in an optical cavity.

Here we propose a different feedback scheme, based on direct photodection rather than homodyne detection. The idea is very simple: whenever the cavity looses a photon, a feedback loop supplies the cavity mode with another photon, through the injection of an appropriately prepared atom. This kind of feedback is naturally suggested by the quantum trajectory picture of a decaying cavity field [11], in which time evolution is driven by the non-unitary evolution operator $\exp\{-\gamma t a^{\dagger} a/2\}$ interrupted at random times by an instantaneous jump describing the loss of a photon. The proposed feedback almost instantaneously "cures" the effect of a quantum jump and is able therefore to minimize the destructive effects of dissipation on the quantum state of the cavity mode.

In more general terms, the application of a feedback loop modifies the master equation of the system and therefore it is equivalent to an effective modification of the dissipative environment of the cavity field. For example, Ref. [12] shows that a squeezed bath [13] can be simulated by the application of a feedback loop based on a quantum non-demolition (QND) measurement of a quadrature of a cavity mode. In other words, feedback is the main tool for realizing, in the optical domain, the so called "quantum reservoir engineering" [14].

The master equation for continuous feedback has been derived by Wiseman and Milburn [9], and, in the case of perfect detection via a single loss source, is given by

$$\dot{\rho} = \gamma \Phi(a \rho a^{\dagger}) - \frac{\gamma}{2} a^{\dagger} a \rho - \frac{\gamma}{2} \rho a^{\dagger} a \ , \tag{1}$$

where γ is the cavity decay rate and $\Phi(\rho)$ is a generic superoperator describing the effect of the feedback atom on the cavity state ρ. Eq. (1) assumes perfect detection, i.e., all the photons leaving the cavity are absorbed by a unit-efficiency photodetector and trigger the cavity loop. It is practically impossible to realize such an ideal situation and therefore it is more realistic to generalize this feedback master equation to the situation where only a fraction $\eta < 1$ of the photons leaking out of the cavity is actually detected and switches on the atomic injector. It is immediate to see that (1) generalizes to

$$\dot{\rho} = \eta \gamma \Phi(a \rho a^{\dagger}) + (1-\eta) \gamma a \rho a^{\dagger} - \frac{\gamma}{2} a^{\dagger} a \rho - \frac{\gamma}{2} \rho a^{\dagger} a \ . \tag{2}$$

Now, we have to determine the action of the feedback atom on the cavity field $\Phi(\rho)$; this atom has to release exactly one photon in the cavity, possibly

regardless of the field state in the cavity. In the optical domain this could be realized using *adiabatic transfer of Zeeman coherence* [15].

2.1 Adiabatic passage in a three level Λ atom

A scheme based on the adiabatic passage of an atom with Zeeman substructure through overlapping cavity and laser fields has been proposed [15] for the generation of linear superpositions of Fock states in optical cavities. This technique allows for coherent superpositions of atomic ground state Zeeman sublevels to be "mapped" directly onto coherent superpositions of cavity-mode number states. If one applies this scheme in the simplest case of a three-level Λ atom one obtains just the feedback superoperator we are looking for, that is

$$\Phi(\rho) = a^\dagger (aa^\dagger)^{-1/2} \rho (aa^\dagger)^{-1/2} a \ , \tag{3}$$

corresponding to the feedback atom releasing exactly one photon into the cavity, regardless the state of the field.

To see this, let us consider a three level Λ atom with two ground states $|g_1\rangle$ and $|g_2\rangle$, coupled to the excited state $|e\rangle$ via, respectively, a classical laser field $\Omega(t)$ of frequency ω_L, and a cavity field mode of frequency ω. The corresponding Hamiltonian is

$$\begin{aligned} &H(t) = \hbar\omega a^\dagger a + \hbar\omega_{eg}|e\rangle\langle e| - i\hbar g(t)\left(|e\rangle\langle g_2|a - |g_2\rangle\langle e|a^\dagger\right) \\ &+ i\hbar\Omega(t)\left(|e\rangle\langle g_1|e^{-i\omega_L t} - |g_1\rangle\langle e|e^{i\omega_L t}\right) \ . \end{aligned} \tag{4}$$

The time dependence of $\Omega(t)$ and $g(t)$ is provided by the motion of the atom across the laser and cavity profiles. This Hamiltonian couples only states within the three-dimensional manifold spanned by $|g_1, n\rangle$, $|e, n\rangle$, $|g_2, n+1\rangle$, where n denotes a Fock state of the cavity mode. Of particular interest within this manifold is the eigenstate corresponding to the adiabatic energy eigenvalue (in the frame rotating at the frequency ω) $E_n = n\hbar\omega$,

$$|E_n(t)\rangle = \frac{g(t)\sqrt{n+1}|g_1, n\rangle + \Omega(t)|g_2, n+1\rangle}{\sqrt{\Omega^2(t) + (n+1)g^2(t)}} \tag{5}$$

which does not contain any contribution from the excited state and for this reason is called the "dark state". This eigenstate exhibits the following asymptotic behavior as a function of time

$$|E_n\rangle \to \begin{cases} |g_1, n\rangle & \text{for} \quad \Omega(t)/g(t) \to 0 \\ |g_2, n+1\rangle & \text{for} \quad g(t)/\Omega(t) \to 0 \end{cases} \tag{6}$$

Now, according to the adiabatic theorem [16], when the evolution from time t_0 to time t_1 is sufficiently slow, a system starting from an eigenstate of $H(t_0)$ will pass into the corresponding eigenstate of $H(t_1)$ that derives from it by continuity. This means that if the atom crossing is such that adiabaticity is satisfied, when

the atom enters the interaction region in the ground state $|g_1\rangle$, the following adiabatic transformation of the atom-cavity system state takes place

$$
\begin{aligned}
&|g_1\rangle\langle g_1| \otimes \sum_{n,m} \rho_{n,m} |n\rangle\langle m| \\
&\to |g_2\rangle\langle g_2| \otimes \sum_{n,m} \rho_{n,m} |n+1\rangle\langle m+1| \\
&= |g_2\rangle\langle g_2| \otimes a^\dagger (aa^\dagger)^{-1/2} \rho (aa^\dagger)^{-1/2} a \ .
\end{aligned} \tag{7}
$$

Roughly speaking, this transformation amounts to a single photon transfer from the classical laser field to the quantized cavity mode realized by the crossing atom, provided that a counterintuitive pulse sequence in which the classical laser field $\Omega(t)$ is time-delayed with respect to $g(t)$ is applied. Figure 1 shows a simple diagram of the feedback scheme, together with the appropriate atomic configuration, cavity and laser field profiles needed for the adiabatic transformation considered.

The quantitative conditions under which adiabaticity is satisfied are obtained from the requirement that the transition from the dark state $|E_n(t)\rangle$ to the other states be very small. One obtains [15, 17]

$$
\Omega_{max}, g_{max} \gg T_{cross}^{-1} \ , \tag{8}
$$

where T_{cross} is the cavity crossing time and Ω_{max}, g_{max} are the two peak intensities.

The above arguments completely neglect dissipative effects due to cavity losses and atomic spontaneous emission. For example, cavity dissipation couples a given manifold $|g_1, n\rangle$, $|e, n\rangle$, $|g_2, n+1\rangle$ with those with a smaller number of photons. Since ideal adiabatic transfer occurs when the passage involves a single manifold, optimization is obtained when the photon leakage through the cavity is negligible during the atomic crossing, that is

$$
T_{cross}^{-1} \gg \bar{n}\gamma \ , \tag{9}
$$

where $\bar{n}$ is mean number of photons in the cavity. On the contrary, the technique of adiabatic passage is robust against the effects of spontaneous emission as, in principle, the excited atomic state $|e\rangle$ is never populated. Of course, in practice some fraction of the population does reach the excited state and hence large values of g_{max} and Ω_{max} relative to the spontaneous emission rate γ_e are desirable. To summarize, the quantitative conditions for a practical realization of the adiabatic transformation (7) are

$$
\Omega_{max}, g_{max} \gg T_{cross}^{-1} \gg \bar{n}\gamma, \gamma_e \ , \tag{10}
$$

which, as pointed out in [15], could be realized in optical cavity QED experiments.

We note that when the adiabaticity conditions (10) are satisfied, then also the Markovian assumptions at the basis of the feedback master equation (2) are

automatically justified. In fact, the continuous feedback theory of Ref. [9] is a Markovian theory derived assuming that the delay time associated to the feedback loop can be neglected with respect to the typical timescale of the cavity mode dynamics. In the present scheme the feedback delay time is due to the electronic trasmission time of the detection signal and, most importantly, by the interaction time T_{cross} of the atoms with the field, while the typical timescale of the cavity field dynamics is $1/\gamma\bar{n}$. Therefore, the inequality on the right of Eq. (10) is essentially the condition for the validity of the Markovian approximation and this *a posteriori* justifies our use of the Markovian feedback master equation (2) from the beginning.

2.2 Properties of the adiabatic transfer feedback model

When we insert the explicit expression (3) of the feedback superoperator into Eq. (2), the feedback master equation can be rewritten in the more transparent form

$$\dot{\rho} = \frac{(1-\eta)\gamma}{2}\left(2a\rho a^{\dagger} - a^{\dagger}a\rho - \rho a^{\dagger}a\right) - \frac{\eta\gamma}{2}\left[\sqrt{\hat{n}}, \left[\sqrt{\hat{n}}, \rho\right]\right] \qquad (11)$$

that is, a standard vacuum bath master equation with effective damping coefficient $(1-\eta)\gamma$ plus an unconventional phase diffusion term, in which the photon number operator is replaced by its square root and which can be called "square root of phase diffusion".

In the ideal case $\eta = 1$, vacuum damping vanishes and only the unconventional phase diffusion survives. As shown in Ref. [18], this is equivalent to say that ideal photodetection feedback is able to transform standard photodetection into a quantum non-demolition (QND) measurement of the photon number. In this ideal case, a generic Fock state $|n\rangle$ is obviously preserved for an infinite time, since each photon lost by the cavity triggers the feedback loop which, in a negligible time, is able to give the photon back through adiabatic transfer. However, the photon injected by feedback has no phase relationship with the photons already present in the cavity and, as shown by (11), this results in phase diffusion. This means that feedback does not guarantee perfect state protection for a generic *superposition of number states*, even in the ideal condition $\eta = 1$. In fact in this case, only the diagonal matrix elements in the Fock basis of the initial pure state are perfectly conserved, while the off-diagonal ones always decay to zero, ultimately leading to a phase-invariant state. However this does not mean that the proposed feedback scheme is good for preserving number states only, because the unconventional "square-root of phase diffusion" is much slower than the conventional one (described by a double commutator with the number operator).

In fact the time evolution of a generic density matrix element in the case of feedback with ideal photodetection $\eta = 1$ is

$$\rho_{n,m}(t) = \exp\left\{-\frac{\gamma t}{2}\left(\sqrt{n}-\sqrt{m}\right)^2\right\}\rho_{n,m}(0)\ , \qquad (12)$$

while the corresponding evolution in the presence of standard phase diffusion is

$$\rho_{n,m}(t) = \exp\left\{-\frac{\gamma t}{2}\left(n-m\right)^2\right\}\rho_{n,m}(0) \ . \tag{13}$$

Since

$$(n-m)^2 \geq \left(\sqrt{n}-\sqrt{m}\right)^2 = \frac{(n-m)^2}{\left(\sqrt{n}+\sqrt{m}\right)^2} \qquad \forall\, n,m \tag{14}$$

each off-diagonal matrix element decays slower in the square root case and this means that the feedback-induced unconventional phase diffusion is slower than the conventional one.

A semiclassical estimation of the diffusion constant can be obtained from the time evolution of the mean coherent amplitude $\langle a(t)\rangle$. In fact, phase diffusion causes a decay of this amplitude as the phase spreads around 2π, even if the photon number is conserved. In the presence of ordinary phase diffusion the amplitude decays at the rate $\gamma/2$; in fact

$$\langle a(t)\rangle = \mathrm{Tr}\left\{a\rho(t)\right\} = \sum_{n=0}^{\infty}\sqrt{n+1}\rho_{n+1,n}(t) \ , \tag{15}$$

and using Eq. (13) one gets

$$\langle a(t)\rangle = e^{-\gamma t/2}\langle a(0)\rangle \ .$$

In the case of the square root of phase diffusion, Eqs. (12) and (15) instead yield

$$\langle a(t)\rangle = \mathrm{Tr}\left\{a(t)\rho(0)\right\} \ , \tag{16}$$

where the Heisenberg-like time evolved amplitude operator $a(t)$ is given by

$$a(t) = \exp\left\{-\frac{\gamma t}{2}\left(\sqrt{aa^\dagger}-\sqrt{a^\dagger a}\right)^2\right\} a \ . \tag{17}$$

In the semiclassical limit it is reasonable to assume a complete factorization of averages, so to get

$$\langle a(t)\rangle = \exp\left\{-\frac{\gamma t}{2}\left(\sqrt{\bar{n}+1}-\sqrt{\bar{n}}\right)^2\right\}\langle a(0)\rangle \ , \tag{18}$$

which, in the limit of large mean photon number $\bar{n}$, yields

$$\langle a(t)\rangle = \exp\left\{-\frac{\gamma t}{8\bar{n}}\right\}\langle a(0)\rangle \ . \tag{19}$$

This slowing down of phase diffusion (similar to that taking place in a laser well above threshold) means that, when the feedback efficiency η is not too low, the "lifetime" of generic pure quantum states of the cavity field can be significantly increased with respect to the standard case with no feedback (see Eq. 11).

3 Optical feedback scheme for the protection of qubits

Photon states are known to retain their phase coherence over considerable distances and for long times and for this reason high-Q optical cavities have been proposed as a promising example for the realization of simple quantum circuits for quantum information processing. To act as an information carrying quantum state, the electromagnetic fields must consist of a superposition of few distinguishable states. The most straightforward choice is to consider the superposition of the vacuum and the one photon state $\alpha|0\rangle + \beta|1\rangle$. However it is easy to understand that this is not convenient because any interaction coupling $|0\rangle$ and $|1\rangle$ also couples $|1\rangle$ with states with more photons and this leads to information losses. Moreover the vacuum state is not easy to observe because it cannot be distinguished from a failed detection of the one photon state. A more convenient and natural choice is *polarization coding*, i.e., using two degenerate polarized modes and qubits of the following form

$$|\psi\rangle = \left(\alpha a_+^\dagger + \beta a_-^\dagger\right)|0\rangle = \alpha|0,1\rangle + \beta|1,0\rangle \ , \tag{20}$$

in which one photon is shared by the two modes [21]. In fact this is a "natural" two-state system, in which the two basis states can be easily distinguished with polarization measurements; moreover they can be easily transformed into each other using polarizers.

Polarization coding has been already employed in one of the few experimental realization of a quantum gate, the quantum phase gate realized at Caltech [5]. This experiment has demonstrated conditional quantum dynamics between two frequency-distinct fields in a high-finesse optical cavity. The implementation of this gate employs two single-photon pulses with frequency separation large compared to the individual bandwidth, and whose internal state is specified by the circular polarization basis as in (20). The conditional dynamics between the two fields is obtained through an effective strong Kerr-type nonlinearity provided by a beam of cesium atoms. This conditional dynamics of the quantum gate has to be unitary with a high degree of accuracy during the operation time, i.e., decoherence has to be negligible; the experiment of Ref. [5] has been performed in the bad cavity limit, in which the main dissipative effects and main source of decoherence is the photon leakage outside the cavity. It is therefore quite natural to see if the atomic feedback scheme described in detail above is able to protect the "flying" qubits of Ref. [5]. To be more specific, we shall not be concerned with the protection of the quantum gate dynamics, but we shall focus on a simpler but still important problem: protecting an unknown input state for the longest possible time against decoherence. We shall therefore consider a single qubit, i.e., a single frequency whose internal state is specified by the polarization.

One has to apply an adiabatic transfer feedback loop as that of Fig. 1 to each polarized mode independently. This can be done using polarization-sensitive detectors (for example a polarized beam splitter and two detectors) and two classical laser fields with opposite circular polarization. In this way one has two similar feedback loops where one polarized mode is involved in the transition

$|g_1\rangle \to |g_2\rangle$, and the other mode participates to the reversed transition. In this way each mode gets a photon with the right polarization. A schematic description of the scheme is given by Fig. 2.

For a quantitative characterization of how the feedback scheme is able to protect an initial pure state we study the fidelity $F(t)$

$$F(t) = \mathrm{Tr}\{\rho(0)\rho(t)\} \tag{21}$$

i.e., the overlap between the final and the initial state $\rho(0)$ after a time t. In general $0 \le F(t) \le 1$. For an initially pure state $|\psi(0)\rangle$, $F(t)$ is in fact the probability to find the system in the initial state at a later time. A decay to an asymptotic limit is given by the overlap $\langle\psi(0)|\rho(\infty)|\psi(0)\rangle$. Since the input state we seek to protect is unknown, the protection capabilities of the feedback scheme are better characterized by the minimum fidelity, i.e., the fidelity of Eq. (21) minimized over all possible initial states. Moreover we shall consider a class of initial states more general than those of Eq. (20), i.e.,

$$|\psi\rangle = \alpha|n,m\rangle + \beta|m,n\rangle\ . \tag{22}$$

Since the two polarized modes evolve independently, one has to solve the master equation (11) to calculate the fidelity. This can be done easily and one gets the following expression for the minimum fidelity

$$F_{min}(t) = \frac{1}{2}\left(e^{-(1-\eta)\gamma t(n+m)} + e^{-\gamma t(n+m-2\eta\sqrt{nm})}\right)\ . \tag{23}$$

In the absence of feedback ($\eta = 0$), this expression becomes $F_{min}(t) = \exp\{-\gamma t(n+m)\}$ showing that in this case, the states most robust against cavity damping are those with the smallest number of photons, $m+n=1$, i.e., the states of the form of Eq. (20). Moreover, in a typical quantum information processing situation, one has to consider small qubit "storage" times t with respect to γ^{-1} so to have reasonably small error probabilities in quantum information storage. Therefore the protection capability of an optical cavity with no feedback applied is described by

$$F_{min}(t) = 1 - \gamma t\ . \tag{24}$$

If we now consider the situation in the presence of feedback (Eq. (23)), it is possible to see that, for fixed, non-unit efficiency η, the best protected state are, as in the no-feedback case, the states with only one photon $\alpha|0,1\rangle + \beta|1,0\rangle$ and therefore the corresponding minimum fidelity for $\eta < 1$ is given by

$$F_{min}(t) = \frac{1}{2}\left(e^{-(1-\eta)\gamma t} + e^{-\gamma t}\right) \simeq 1 - \gamma t\left(1 - \frac{\eta}{2}\right)\ . \tag{25}$$

This shows that feedback increases the "lifetime" of a generic qubit state with respect to the no-feedback case, even if, in this non-ideal case, one has a scaling of the error probability by a factor $(1-\eta/2)$ only.

It is interesting to consider the ideal case $\eta = 1$, even if it is not realistic, because in this case the situation changes qualitatively. In fact Eq. (23) becomes

$$F_{min}(t) = \frac{1}{2}\left(1 + e^{-\gamma t(\sqrt{n}-\sqrt{m})^2}\right) \ . \tag{26}$$

so that it is easy to see that in this case it becomes convenient to work with a large number of photons and that the best protected qubit states are those of the form

$$|\psi\rangle = \alpha|n, n+1\rangle + \beta|n+1, n\rangle \qquad n \gg 1 \tag{27}$$

whose corresponding minimum fidelity is

$$F_{min}(t) \simeq \frac{1}{2}\left(1 + e^{-\gamma t/4n}\right) \simeq 1 - \frac{\gamma t}{8n} \ .$$

Therefore, in the ideal photodetection case and using qubits of the form of (27), the probability of errors in the storage of quantum information can be made arbitrarily small.

The feedback method proposed here to deal with decoherence in quantum information processing is different from most of the proposals made in this research field, which are based on the so called quantum error correction codes [22]. In our case, feedback allows a physical control of decoherence, through a continuous monitoring and eventual correction of the dynamics and in this sense our approach is similar in spirit to the approach of Ref. [23, 24]. Quantum error correction is instead a way to use *software* to preserve linear superposition states. Essentially in these approaches the entangled superposition state of l qubits is "encoded" in a larger number of qubits n, so that, assuming that only a fraction of qubits t/n decoheres, it is possible to reconstruct the original state with a suitable decoding procedure. However, due to existence of a lower (quantum Hamming) and an upper (quantum Gilbert-Varshamov) bound for the "code rate" l/n [25], these quantum error correction codes can be applied and become efficient only at sufficiently small probability of error t/n. For this reason, even if under realistic conditions our feedback scheme achieves only a moderate reduction of the error probability, it could be useful when used in *conjunction* with quantum error correction techniques. The feedback scheme would realize the preliminary reduction of the error probability, which is necessary for an optimal implementation of efficient error correction schemes.

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